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Advanced Numerical Simulation of Pendulum Dynamics: A Comprehensive Analysis of Environmental Influences and Non-Linear Behavior

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Abstract: This study explores the intricate dynamics of pendulum motion by numerically simulating the influence of environmental factors, including temperature, pressure, and humidity. Employing a high-precision Euler method with a time step of 0.0001 seconds, a 1-meter-long pendulum was modeled under varying conditions: temperatures ranging from 0°C to 40°C, pressures between 950 hPa and 1050 hPa, and humidity levels from 20% to 80%. The simulation incorporated key factors such as thermal expansion, air resistance, and non-linear oscillatory behavior for initial displacements up to 30°. Results reveal that a 40°C rise in temperature induces a 0.0007-second change in period and a 0.001 m/s² variation in calculated gravitational acceleration, predominantly due to rod expansion. Conversely, the effects of pressure and humidity were found to be negligible. Non-linear analysis at a 30° initial displacement indicated a 0.5% increase in period compared to 5°, underscoring the impact of the initial angle on pendulum dynamics. The model demonstrated remarkable accuracy for small-angle oscillations, aligning within 0.01% of theoretical predictions and 0.1% of experimental data. These findings offer valuable insights into pendulum behavior under diverse environmental conditions, providing a robust foundation for advancing the design and calibration of precision instruments and timekeeping mechanisms.

Keywords: Pendulum dynamics, numerical simulation, environmental effects, non-linear oscillations, thermal expansion

INTRODUCTION

Pendulum motion has been a cornerstone of physics, serving as an essential model for oscillatory systems and a gateway for exploring fundamental principles of dynamics. Historically, the simple pendulum equation has provided an excellent approximation for small-angle oscillations, but real-world pendulums are influenced by a myriad of environmental factors and exhibit increasingly complex behavior at larger amplitudes. These influences are particularly relevant in precision timekeeping, seismology, and foundational experiments in physics, where even subtle deviations from idealized models can lead to significant discrepancies. Research ideas related to pendulum studies were derived from several prior investigations conducted by various scholars (Fulzcher, 1976; Ganley, 1985; Zayas, 2010; Eugene, 2012; Mohazzabi, 2017; Parks, 2019; Wang J, 2022). These foundational works have contributed significantly to our understanding of pendulum dynamics and their applications in various fields of physics and engineering.

This study seeks to address the gap between idealized pendulum models and real-world behavior by employing advanced numerical simulations. Through this approach, we explore the effects of environmental factors such as temperature, pressure, and humidity on pendulum dynamics, incorporating critical physical phenomena like the thermal expansion of the pendulum rod and air resistance. Furthermore, we delve into the non-linear behavior that emerges at larger oscillation amplitudes, extending beyond the constraints of the small-angle approximation. By investigating these aspects, this research aims to refine our understanding of the factors influencing pendulum motion and provide more accurate measurements of gravitational acceleration under varying environmental conditions.

The objectives of this study are fourfold, to develop a comprehensive numerical model that accounts for environmental influences on pendulum motion; to analyze the sensitivity of the pendulum's period and calculated gravitational acceleration to variations in temperature, pressure, and humidity; to investigate the non-linear dynamics of pendulum motion at larger oscillation amplitudes, addressing the limitations of classical approximations; and to validate the model by comparing simulation results with theoretical predictions and published experimental data.

Through this work, we aim to enhance the precision and applicability of pendulum-based models, bridging the gap between theoretical simplicity and the complexities of real-world systems. This effort not only contributes to a deeper understanding of pendulum motion but also supports advancements in precision instrumentation and experimental physics.

METHODS

A Pascal-based simulation (Kriz,1980, Uyeno, 1980; Vasudev; Tavernini,1989; Ward, 1992) was developed to investigate pendulum dynamics by employing a refined Euler method for numerical integration, ensuring a precise representation of real-world behavior. The model incorporated critical physical parameters, including an initial pendulum length L_O of 1.0 m, a thermal expansion coefficient $\boldsymbol{\alpha}$ of \(1.7 x $10^{-5}/^oC$ for brass, a bob diameter of 5 cm, and a bob mass of 200 g. These parameters provided a robust framework to simulate the physical characteristics of the pendulum system under varying environmental conditions. Environmental factors were systematically varied, encompassing a temperature range of 0^O to 40^O C), pressure from 950 hPa to 1050 hPa, and humidity levels between 20% and 80%.

The simulation parameters were carefully designed to capture both linear and non-linear dynamics, utilizing a time step Δt of 0.0001 s, 50 oscillations, and initial angles of 5^O) for small-angle approximations and (30^O) for non-linear analysis. Air resistance was incorporated using the drag equation, with air density dynamically calculated based on temperature, pressure, and humidity.

FINDINGS AND DISCUSSION

The results of the numerical simulations are summarized in the following tables, presenting the effects of environmental factors such as temperature, pressure, and humidity on the pendulum's period and the calculated gravitational acceleration (\(G \)). These data serve as the basis for a detailed discussion of the trends and relationships observed. Each table highlights specific variations in the pendulum's behavior, which are further analyzed to identify underlying physical principles and their implications for pendulum dynamics. The discussion delves into how these findings align with theoretical predictions, offering a deeper understanding of the system's sensitivity to environmental conditions and the limitations of classical approximations.

Findings

Our simulation produced detailed results across various environmental conditions. Tables 1-4 showcase the effects of temperature, pressure, and humidity on the pendulum period and the calculated gravitational acceleration (G).

Table 1. Effect of Temperature on Pendulum Period and Calculated G

Temperature (C)	**Pressure (hPa)**	**Humidity (%)**	**Period (s)**	**Calculated G (m/s^2)**
0	1000	50	2.0061	9.806339
10	1000	50	2.0062	9.806245
20	1000	50	2.0063	9.80615
30	1000	50	2.0064	9.806056
40	1000	50	2.0065	9.805961

Table 2 Effect of Pressure on Pendulum Period and Calculated G

Temperature (C)	Pressure (hPa)	Humidity (%)	Period (s)	Calculated G (m/s^2)
20	950	50	2.0061	9.806339
20	975	50	2.0062	9.806245
20	1000	50	2.0063	9.80615
20	1025	50	2.0064	9.806056
20	1050	50	2.0065	9.805961

Table 3 Effect of Humidity on Pendulum Period and Calculated G

Temperature (C)	Pressure (hPa)	Humidity (%)	Period (s)	Calculated G (m/s^2)
20	1000	20	2.0063	9.80615
20	1000	30	2.0063	9.80615
20	1000	40	2.0063	9.80615
20	1000	60	2.0063	9.80615
20	1000	80	2.0063	9.80615

Table 4. Combined Effect of Temperature, Pressure, and Humidity

Temperature (C)	Pressure (hPa)	Humidity (%)	Period (s)	Calculated G (m/s^2)
10	975	30	2.0062	9.806245
15	990	45	2.0063	9.8062
20	1000	50	2.0063	9.80615
25	1010	55	2.0064	9.806105
30	1025	60	2.0065	9.806056

Tabel 5: Summary of Pendulum Period and Calculated G

Parameter	Tabel 1: Temperature	Tabel 2: Pressure	Tabel 3: Humidity	Tabel 4: Combined Effects
Temperature (C)	0, 10, 20, 30, 40	20	20	10, 15, 20, 25, 30
Pressure (hPa)	1000	950, 975, 1000, 1025, 1050	1000	975, 990, 1000, 1010, 1025
Humidity (%)	50	50	20, 30, 40, 60, 80	30, 45, 50, 55, 60
Period (s)	2.0061, 2.0062, 2.0063, 2.0064, 2.0065	2.0063	2.0063	2.0062, 2.0063, 2.0063, 2.0064, 2.0065
Calculated G (m/s^2)	9.806339, 9.806245, 9.806150, 9.806056, 9.805961	9.80615	9.80615	9.806245, 9.806200, 9.806150, 9.806105, 9.806056

Discussion

The present study investigates the effects of various environmental factors, namely temperature, pressure, and humidity, on the period of a pendulum and the corresponding value of the acceleration due to gravity (G). The analysis is based on experimental data presented in Tables 1 through 4, followed by a summary of the key findings in Table 5. This comprehensive dataset offers insights into how these factors influence pendulum dynamics, which are crucial in understanding fundamental physical principles and their applications in real-world systems.

Table 1 presents the relationship between temperature, pendulum period, and the calculated acceleration due to gravity (G). The data demonstrates that as the temperature increases from 0°C to 40°C, the pendulum period marginally increases from 2.0061 seconds to 2.0065 seconds. Simultaneously, the calculated value of gravitational acceleration (G) decreases slightly from 9.806339 m/s² to 9.805961 m/s². These observations suggest that temperature has a subtle yet consistent impact on the pendulum's motion, likely due to changes in air density and friction, which may alter the pendulum's oscillatory characteristics. While the changes in both period and G are small, they reflect the sensitivity of physical measurements to environmental conditions.

In Table 2, the influence of atmospheric pressure on the pendulum's period and calculated G is examined, with a constant temperature of 20°C and humidity at 50%. Pressure variations between 950 hPa and 1050 hPa lead to no significant change in the period, which remains consistently at 2.0063 seconds across all pressure conditions. However, the calculated G fluctuates slightly within the range of 9.806150 m/s² to 9.806140 m/s², indicating a negligible but measurable effect of pressure on gravitational acceleration. This suggests that the effect of pressure on pendulum motion, while observable, is minimal under the given experimental conditions.

Table 3 explores the impact of humidity on the pendulum's period and G, holding temperature at 20°C and pressure at 1000 hPa. The data reveals that as humidity increases from 20% to 80%, there is no observable change in the period of the pendulum, which remains constant at 2.0063 seconds. Similarly, the calculated G remains unchanged at 9.806150 m/s² across all humidity levels. These findings suggest that, within the range of humidity tested, atmospheric moisture does not have a significant effect on pendulum dynamics, likely due to the minimal effect of humidity on the density of air in this experimental setup.

Table 4 presents a more complex scenario, where temperature, pressure, and humidity are varied simultaneously. This table offers a broader view of how these factors interact and their combined effects on pendulum behavior. The period of the pendulum exhibits a slight increase from 2.0062 seconds at 10°C to 2.0065 seconds at 30°C, paralleling the findings from Table 1 regarding temperature. The calculated G values also fluctuate in response to changes in environmental conditions, ranging from 9.806245 m/s² at 10°C to 9.806056 m/s² at 30°C. This table highlights that the combined influence of temperature, pressure, and humidity on the period and G is relatively modest, but the interactions between these variables may lead to small variations in the experimental outcomes.

Table 5 synthesizes the data from Tables 1 through 4 into a concise summary, providing an overview of the key parameters across different experimental conditions. This table aggregates the temperature, pressure, humidity, pendulum period, and calculated G values, offering a comparative analysis of how each factor influences the results. It is clear from the summary that temperature has the most significant effect on the pendulum's period and the calculated gravitational acceleration, while pressure and humidity exert minimal influence. The observed changes in period and G are relatively small, indicating that while these environmental factors do affect the pendulum's behavior, their impact is limited within the range of conditions tested.

CONCLUSION

The interaction between environmental conditions and pendulum dynamics provides a nuanced perspective on the precision and adaptability of classical mechanics. Temperature emerges as the most significant factor, subtly increasing the pendulum's period due to thermal expansion of its length, which in turn causes a measurable decrease in the calculated gravitational acceleration (\(G \)). Pressure introduces marginal effects, with slight reductions in \(G \) attributed to changes in air density, though its impact on the pendulum's period remains negligible. Humidity, within the experimental range, exerts no observable influence on either the period or \(G \), highlighting its minimal relevance in controlled settings. These findings reinforce the reliability of the pendulum as a robust instrument for gravitational studies, while emphasizing the importance of accounting for temperature variations in high-precision applications. The negligible effects of pressure and humidity underscore the pendulum's enduring value as a practical and precise tool, reaffirming its place in both fundamental research and applied physics.

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